Outline for: An Empirical Equilibrium Model of the U.S. Labor Market 1972-2000

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This is not a formal paper, but an outline of the basic model and summary of preliminary results.

Model of Individual Consumer

Utility function:

$$U(S,C,H) = \sum_{a=0}^{A_r-1} \delta^a \left(\frac{c_a^{\gamma}}{\gamma} - A \frac{h_a^{\eta}}{\eta} \right) + \sum_{a=A_r}^{A_d} \delta^a \frac{c_a^{\gamma}}{\gamma} + v_S \tag{1}$$

where c_a is consumption, A_r is age at retirement, A_d is age at death, v_S is nonpecuniary benefits of schooling, and γ, A, η and δ are parameters of the utility function. In the utility function S is school and C and H are the paths of consumption and labor supply.

People receive wage W_a in the market but must pay a cost $(1 - \pi_a)$ per hour to work.¹ Home production is linear at rate Y_a . Total hours worked

$$h_a = h_{ma} + h_{ha} \tag{2}$$

where h_{ma} is hours in the market, and h_{ht} is hours working at home.

We get a budget constraint for consumption and labor supply

$$\sum_{a=0}^{A_d} \frac{1}{R_a} c_a = \sum_{a=S}^{A_d} \frac{1}{R_a} \max\left\{h_{ma} W_a \pi_a, h_{ha} Y_a\right\} + K_0 \tag{3}$$

where $1/R_a$ is the price of consumption at age a and K_0 is initial assets. Since home production is linear, individuals will choose to specialize between work at home and work on the job. We are assuming (for now) that once someone enters the labor force their is full certainty about the path of wages, home production, and interest rates.

However, we relax this assumption for schooling decisions. Instead we assume that they know the distribution of wages. Let $V(S, K_0)$ be the realized value function the individual would receive if they had chosen school level S (assuming they choose consumption and labor supply optimally). That is

$$V(S,K) = \max_{C,H} U(S,C,H)$$

subject to (3)

¹Ideally one would like this cost to be a "fixed" cost and not depend on hours worked. However, this would lead to a much more complicated model to solve. Given that the intertemporal elasticity of substitution for hours worked is small it is unlikely that this distinction is important in practice.

We will relax this perfect certainty assumption for schooling decisions. Instead we assume that they know the distribution of wages, thus a student chooses to attend college if

$$E[V(1, K_0))] \ge E[V(0, K_0)]$$

Aggregation and Demand for Skill

This is mainly the same as in our previous papers. At a point in time we aggregate "high school" human capital by integrating over all of the high school workers who work in the market at time t. At time t, their time is rented at rate P_t^h . The wage for individual i at time t is his human capital H_{it}^h times the rental rate,

$$W_{it}^h = P_t^h H_{it}^h$$

Similarly we aggregate college workers and they rent their human capital H_{it}^c at rate P_t^c so that their wages can be written as

$$W_{it}^c = P_t^c H_{it}^c$$

The main difference is that I shut down the physical capital market so it is essentially the open economy version of the model.

We solve for a competitive equilibrium with a constant returns to skill aggregate production function. Thus, the rental rates on skills are just their marginal products.

We will discuss aggregation and the production function below.

Overview of Simulation/Estimation

A major goal of this work is trying to completely integrate estimation and simulation so everything is mutually compatible. The general algorithm is the following:

- 1. Estimation of intertemporal elasticity of labor supply
- 2. Estimate parameters of wage equation, home production and path of price paths for college and high school educated workers

- 3. Construct college attendance probabilities for cohorts prior to initial cohort as well as cohort size for all cohorts
- 4. Estimate parameter A, the tastes for leisure
- 5. Given actual prices and actual college attendance decisions, estimate parameters of schooling decision
- 6. Iterate on the following algorithm until convergence in prices:
 - Begin with a set of prices (using actual prices projected into future at first iteration)
 - Given these prices, simulate the supply of capital and labor supply to the model for all cohorts at all time periods
 - Using the simulated supply of factors and actual prices, estimate the parameters of the aggregate production function
 - Now plug the full set of supply parameters into the production function to get a new set of prices
- 7. Once the model has been estimated we will use it to simulate the effects of several policies

Estimation of Intertemporal Elasticity of Labor Supply

Given that there is no uncertainty in this model, η can be estimated using a standard fixed effects strategy. For individuals who work

$$\log (h_a) = \beta_0 + \frac{1}{\eta - 1} \log (W_a) + \log (\lambda)$$

where λ is the marginal utility of income.

We deal with measurement error in labor supply by using annual earnings/annual hours as our wage measure, but using the hourly wage last week measure as an instrument (actually we use the wage measure in the year of the survey, a year before, and a year later).

Estimation of prices, wage equation, and home production

We assume that for $s \in \{c, h\}$ and for individual *i* at time *t*, human capital takes the form

$$\log(H_{it}^s) = Z_{it}'\gamma^s + \theta_i^s + u_{it}^s$$

where Z_{it} is a set of characteristics of individual *i* at time *t*, θ_i is a random effect (which in practice will have a discrete distribution), and u_{it} is an iid $N(0, \sigma_{u^s}^2)$. Thus for those who work, we also observe wages which are

$$\log(W_{it}^s) = \log(P_t^s H_{it}^s)$$
$$= \log(P_t^s) + Z_{it}\gamma^s + \theta_i^s + u_{it}^s$$

The probability of working in a period is

$$\Pr\left(W_{it}^s > \frac{Y_{it}}{\pi_{it}} \mid Z_{1i}, Z_{2i}, X_{it}, \theta_i\right).$$

Since we can not tell the difference between Y_{it} and π_{it} we will combine not try to distinguish between them but will assume that

$$\log\left(\frac{Y_{it}^s}{\pi_{it}^s}\right) = \rho_t^{*s} + Z_{it}^{\prime}\beta^{*s} + \alpha^{*s}\theta_i^s + b_u^{*s}u_{it}^s + \varepsilon_{it}^s$$

where ε_{it}^s is $N(0, \sigma_{\varepsilon^s}^2)$ and ε is independent of u^2 . Note that this has been written as if the covariates in both the wage and selection equations are the same. We will allow for exclusion restrictions by thinking of γ^s as restricted to zero for some covariates.

$$\begin{aligned} &\Pr\left(\frac{Y_{it}}{\pi_{it}} > W_{it}^{s} \mid Z_{1i}, Z_{2i}, X_{it}, \theta_{i}\right) \\ &\equiv &\Pr\left(\rho_{t}^{*s} + Z_{it}'\beta^{**} + \alpha^{**}\theta_{i}^{s} + b_{u}^{**}u_{it}^{s} + \varepsilon_{it}^{s} > \log(P_{t}^{s}) + Z_{it}\gamma^{s} + \theta_{i}^{s} + u_{it}^{s} \mid Z_{1i}, Z_{2i}, X_{it}, \theta_{i}\right) \\ &= &\Phi\left(\frac{\rho_{t} - \frac{1}{\sigma_{\varepsilon}}\log(P_{t}^{s}) + Z_{it}'\beta^{s} + \alpha^{s}\theta_{i}^{s}}{\sqrt{1 + b_{u}^{s^{2}}\sigma_{u}^{s^{2}}}}\right) \end{aligned}$$

where

$$\beta^s = \frac{\beta^{*s} - \gamma^S}{\sigma_{\varepsilon}}$$

²Since the error terms are nomally distributed this last assumption can be made without loss of generality since b_u^{*s} is a free parameter.

$$\begin{aligned} \alpha^{s} &= \frac{\alpha^{*s} - 1}{\sigma_{\varepsilon}} \\ b_{u}^{s} &= \frac{b_{u}^{*s} - 1}{\sigma_{\varepsilon}} \\ \rho_{t} &= \frac{\rho_{t}^{*s}}{\sigma_{\varepsilon}} \end{aligned}$$

We then estimate the model in this form by nonparametric maximum likelihood approximating the distribution of θ_i . At this stage we do not attempt to estimate σ_{ε} , but approximate ρ_t^{*s} using a polynomial in time. Note that we are treating σ_{ε} somewhat differently than σ_u . This is simply because when we estimate the reduced form model some scale normalization is necessary and we chose this particular one.

To do a good job of estimating ideally one would have variables that enter the selection equation but not the wage equation. For this we use family background characteristics at the current time, controlling for other family background variables. We are using a random effect with a fairly long panel so this effect will be identified in large part not from levels of the family background variables, but from changes.

The results from this procedure without heterogeneity (i.e. θ_i) are presented in Tables 1a for men and Table 1b for women. The results with heterogeneity are presented in Tables 2a and 2b. The variables presented in the table in the above notation are $\beta_s, b_u^s, \alpha^s, \gamma^s$, and σ_u^s . Most of the variables take the expected signs and magnitudes.

There are two problems with using the parameters estimated above. First, we only have price data for the period in the NLSY and separating the price data from the experience coefficient is problematic as well. Second, we can not do a good job estimating the experience profile since we only have young workers. This is particularly problematic for retirement. We thus augment these estimates by estimating prices and experience profiles using the CPS.

We are explicitly assuming throughout that cohorts are ex-ante identical. Thus we can simulate the model from the NLSY and compare to aggregate effects from the CPS. This is done using a three step method, first estimate the reduced form selection equation, then estimate the wage equation, then finally estimate the parameters of home production from a structural probit. From the CPS we can estimate the probability of working conditional on schooling and experience at any point in time. Call this estimated value \hat{P}_{stX} . We can also estimate the expected log wage conditional on working for any schooling/experience group at a point in time, call this \widehat{W}_{stX} . We know that

$$\operatorname{plim}\left\{\widehat{P}_{stX}\right\} = \operatorname{plim}\left\{\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{K}\Phi\left(\frac{\rho_t - \frac{1}{\sigma_{\varepsilon}}\log(P_t^s) + Z_{it}'\widehat{\beta}^s + \widehat{\alpha}^s\widehat{\theta}_i^s}{\sqrt{1 + b_u^{s^2}\sigma_u^{s^2}}}\right)\widehat{\mu}_j^s\right\}$$

 $\operatorname{plim}\left\{\widehat{W}_{stX}\right\} = \operatorname{plim}\left\{\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{K}\left(\log(P_{t}^{s}) + Z_{it}^{\prime}\widehat{\gamma}^{s} + \widehat{\theta}_{i}^{s} + E\left(u_{it}^{s} \mid W_{it}^{s} > \frac{Y_{it}^{s}}{\pi_{it}^{s}}, Z_{1i}, Z_{2i}, X_{it}, \widehat{\theta}_{i}\right)\right)\widehat{\mu}_{j}^{s}\right\}$

where "hats" on the right hand side represent parameters estimated in the NLSY, N is the number of observations in the NLSY, and $\{\hat{\mu}_j^s, \hat{\theta}_j^s\}$ j = 1, ..., K is the estimated distribution of θ_i^s . We estimate the parameters in this model in three steps

- We first estimate the reduced form parameters in the first equation by nonlinear least squares. That is we estimate the β^s coefficients on the intercept, on experience, and on experience squared as well as $\rho_t \frac{1}{\sigma_{\varepsilon}} \log(P_t^s)$ for each period.
- We then use these reduced form variables to construct the mills ratio term

$$E\left(\varepsilon_{it}^{s} \mid W_{it}^{s} > Y_{it}, Z_{1i}, Z_{2i}, X_{it}, \theta_{i}\right)$$

and estimate the price changes and the experience profile by nonlinear least squares.

Finally we use our estimates of ρ_t - ¹/_{σε} log(P^s_t) and log(P^s_t) to form estimates of σ_ε and the path of ρ_t assuming that ρ_t is quadratic in time with intercepts that are schooling specific (the linear and quadratic terms are constrained to be the same for high school and college graduates).

The estimated prices for college and high school are presented in Figure 1. Note that both are normalized to 1 in 1963.

Constructing the Probability of College Attendance and Cohort Sizes

Ideally one would construct one measure of schooling and use it throughout. Unfortunately there are a number of complications. The first is that the educational question in the CPS have changed. A second is that the fraction of people attending college tends to raise with a cohort as it ages. This is part real in that people go back to college, but other factors seem to be important as well as people's answer to the question seems to change as they age. To deal with this problem we measure schooling attendance for a cohort as the fraction of individuals age 24 or 25 that have ever attended college. This will result in two additional problems. First, when we run wage regressions we need to use older workers as well so we must use reported schooling for them which may be different than what they would have reported at age 24. Second, we need to project schooling backward for older cohorts so that we can simulate the stock of human capital at a point in time. Clearly we do not observe college attendance at age 24 or 25 for someone born in 1920 so we have to measure it at an older age. We impute schooling for them in a smooth way. We have experimented substantially with different methods of dealing with schooling and the basic results are not terribly sensitive to different reasonable ways of approximating schooling. Denote the estimated probability of attending college for a cohort b as PC_b . The results of this procedure are presented in Figure 2.

For each cohort that is alive during the simulation period we also estimate the size of that cohort. We do this by averaging cohort size over all years of the CPS for which we have data on that cohort as long as they are between 18 and 64. These sizes are presented in Figure 3.

Getting from the Micro Data to Macro Simulation

The next step is to use the evidence from the micro data in the general equilibrium model. The ultimate goal of the work is to perform welfare analysis of various policies so we need the model to be consistent with the distribution of earnings in the united states. With this goal in mind we take a random subsample of the N observations in the NLSY data. Denote this subsample as N_m which in practice is 100, but we will experiment with different values. To save on notation we continue to index these observations by i, but now we assume that $i = 1, ..., N_m$ indexes the subsample of the data.

An additional problem is in separating Y_{it} from π_{it} . For the participation decision they are equivalent and a decrease in home production is identical to an increase in costs of going to work. However, as we look across time at changes in utility it makes a large difference. In the model above, the increased participation rate among women will be attributed to decreases in ρ_t . Whether that is due to a decrease in the cost of going to work or a decrease in home production have opposite welfare effects on the to the utility of the woman. Properly attributing this is important to getting the college decision correct. At this point we deal with this in an ad-hoc manner. In order to force identification of schooling decisions to come from price changes rather than our handling of home production, we assume that the effects go through both Y_{it} and π_{it} and exactly offset so the utility difference between work and home production over time is driven exclusively by labor market skill prices. Thus the participation decision is determined in the manner above, but in figuring utility (and hours of work) we treat the wage if working as the wage and the "wage at home" as fixed over time.

Estimating the Taste for Work

We estimate the utility value A to be that value that equates hours of worked for young workers observed in the CPS.

Estimating the Parameters of the Schooling Decision

We are currently estimating the model assuming myopic expectations, but that is something that we will play around with.

Individuals *i* are assumed to know the value of their observables, but not the value of their random permanent component or the transitory components of income. By integrating over all of the possible outcomes we can calculate the expected utility from consumption and leisure conditional on schooling EU_{ib}^h and EU_{ib}^c . We perform this integration through a monte carlo simulation where we draw a sample of $(u_{\ell t}^s, \varepsilon_{\ell t}^s)$ letting ℓ index the monte carlo draw.

Assuming that the tastes for schooling $(v_c - v_h)$ are normally distributed with mean μ_v and standard deviation σ_v ,

$$\Pr(S=1|i,b) = \Phi\left(\frac{EU_{ib}^c - EU_{ib}^h + \mu_v}{\sigma_v}\right),\,$$

where Φ is the cdf of a standard normal. We then estimate μ_v and σ_v by nonlinear least

squares

$$\sum_{b=B_{\ell}}^{B_u} \left(\frac{1}{N_m} \sum_{i=1}^{N_m} \Phi\left(\frac{EU_{ib}^c - EU_{ib}^h + \mu_v}{\sigma_v} \right) - PC_b \right)^2.$$

This is done separately form men and for women.

The fit from this estimation process are presented in Figure 4. One can see that the model fits the data very well.

Simulation of Model and Estimation of Aggregate Production Function

Next we simultaneously simulate and estimate the model in an attempt to get the simulated model as close as possible to the actual data.

- Given a set of prices we can simulate out the factors in the model using the estimated parameters and model above. That is given a vector of prices that the individual will face during the lifecycle we can compute their schooling and labor supply decisions.

$$\overline{H_{itb}^{s}} = E\left(e^{Z_{1i}\beta^{s} + \theta_{i}^{s} + \beta_{2}^{s}(t-b) + \beta_{3}^{s}(t-b)^{2} + \varepsilon_{it}^{s}} \mid P_{t}^{s}H_{it} > Y_{it}, Z_{1i}, Z_{2i}\right).$$

Men and women are assumed to be perfect substitutes. However women enter at a fraction of men that is allowed to change over time. We then aggregate as

$$H_{st} = \frac{1}{A_r} \sum_{b=t}^{t-A_r+1} \frac{1}{N_m} \sum_{i=1}^{N_m} \overline{H_{itb}^s}$$

• Now consider estimation of the aggregate production function. Using notation similar to HLT we assume that

$$F(K, H_1, H_0; t) = bK^{\theta} \left(a_t H_1^{\rho} + (1 - a_t) H_0^{\rho} \right)^{\frac{1 - \theta}{\rho}}$$

Assume (as in HLT) that

$$\log\left(\frac{1+a_t}{a_t}\right) = \delta_0 + \delta_1 t$$

 then

$$\log\left(\frac{p_{1t}}{p_{0t}}\right) = \delta_0 + \delta_1 t + (\rho - 1) \log\left(\frac{H_{1t}}{H_{0t}}\right).$$

We estimate this model by OLS. The fit of this model is presented in Figure 5. This gives an elasticity of substitution of 1.11.

The other parameters are determined to let the initial prices take the values we choose.

• In simulating the model we assume that the productivity shock lasts for 30 years and then stops. We then construct the prices from the first order conditions for the production function. We will play around with this extensively. The results of this simulation are presented in Figures 6-8.

Estimat	Estimated from NLSY							
No Heterogeneity								
Men Only								
	High S	School	Col	College				
	Stay	Log	Stay	Log				
	Home	Wage	Home	Wage				
Black	0.313	-0.029	0.539	-0.026				
$\operatorname{Hispanic}$	0.105	-0.026	-0.023	-0.101				
AFQT Score	-0.016	0.006	-0.017	0.008				
Urban Area	-0.203	0.060	-0.257	0.071				
Lives in SMSA	0.055	0.082	0.081	0.186				
West	-0.012	0.072	-0.104	0.097				
Northest	-0.085	0.125	-0.425	0.178				
South	-0.332	-0.020	-0.703	0.046				
Potential Experience	-0.102	0.090	0.030	0.082				
Pot. Exp. Squared	0.421	-0.317	-0.195	-0.255				
Year	0.011	-0.033	-0.059	-0.004				
Year Squared	0.003	0.001	0.006	0.000				
Married by 30	0.457	0.362	1.530	0.113				
Age Married	-0.030	-0.009	-0.067	0.002				
Number Children at 30	0.025	-0.019	-0.177	-0.014				
Any Children at 30	-0.251	0.123	0.199	0.127				
Age First Birth	0.002	0.000	-0.001	-0.002				
$\operatorname{Constant}$	-1.217	1.249	-1.766	1.349				
Married	-0.266		-0.438					
Any children	-0.262		-0.229					
Number of children	0.084		0.098					
b_u	-0.097		0.858					
σ_u		0.542		0.577				

Table 1aParameters for Wage and Home Production EquationsEstimated from NLSYNo Heterogeneity

Estima	ted from	NLSY						
No Heterogeneity								
Women Only								
	High S	School	College					
	Stay	Log	Stay	Log				
	Home	Wage	Home	Wage				
Black	-0.159	-0.029	-0.284	-0.026				
$\operatorname{Hispanic}$	-0.100	-0.026	-0.006	-0.101				
AFQT Score	-0.018	0.006	-0.010	0.008				
Urban Area	-0.047	0.060	0.035	0.071				
Lives in SMSA	-0.134	0.082	-0.128	0.186				
West	-0.057	0.072	-0.132	0.097				
Northest	0.026	0.125	-0.075	0.178				
South	-0.213	-0.020	-0.285	0.046				
Potential Experience	-0.048	0.090	0.048	0.082				
Pot. Exp. Squared	0.079	-0.317	-0.208	-0.255				
Year	-0.011	-0.033	-0.133	-0.004				
Year Squared	0.001	0.001	0.005	0.000				
Married by 30	-0.408	0.362	-0.669	0.113				
Age Married	-0.001	-0.009	0.017	0.002				
Number Children at 30	0.116	-0.019	0.086	-0.014				
Any Children at 30	-0.333	0.123	0.079	0.127				
Age First Birth	0.008	0.000	-0.005	-0.002				
Constant	-0.486	1.249	-0.931	1.349				
$\operatorname{Married}$	0.026		0.210					
Any children	0.579		0.586					
Number of children	0.098		0.160					
b_u	-0.129		-0.135					
σ_u		0.530		0.530				

Table 1bParameters for Wage and Home Production EquationsEstimated from NLSYNo Heterogeneity

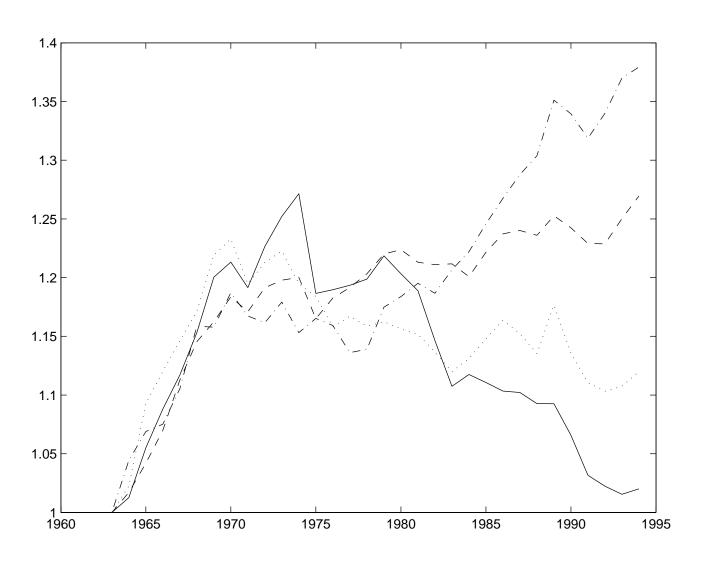
	Estimated	from NLSY			
	Allowing For	Heterogeneit	у		
	Ν	ſen			
	High	School	Col	lege	
	Stay	Log	Stay	Log	
	Home	Wage	Home	Wage	
Black	0.412	-0.045	0.629	-0.013	
Hispanic	0.155	-0.001	-0.207	-0.067	
AFQT Score	-0.016	0.007	-0.014	0.007	
Urban Area	-0.164	0.056	-0.286	0.036	
Lives in SMSA	0.029	0.059	0.117	0.118	
West	-0.072	0.066	0.025	0.100	
Northest	-0.087	0.123	-0.184	0.160	
South	-0.302	-0.005	-0.445	0.035	
Potential Experience	-0.110	0.091	0.025	0.057	
Pot. Exp. Squared	0.515	-0.326	-0.161	-0.160	
Year	-0.002	-0.037	-0.105	0.003	
Year Squared	0.003	0.002	0.009	0.000	
Married by 30	0.257	0.340	1.388	-0.080	
Age Married	-0.021	-0.008	-0.054	0.007	
Number Children at 30	0.025	-0.039	-0.127	-0.020	
Any Children at 30	-0.244	0.150	-0.004	0.149	
Age First Birth	0.000	0.000	-0.003	-0.001	
Constant	2.033	-0.521	1.378	-0.426	
Married	-0.127		-0.652		
Any children	-0.242		-0.024		
Number of children	0.073		0.120		
b_u	0.091		-0.365		
lpha	-2.065		-1.865		
σ_u		0.449		0.422	
	Distribution o	f Heterogenei	ty		
	High School College				
	Mass Point	Probability	Mass Point	Probability	
	0.000	0.014	0.000	0.006	
	0.044	0.000	0.013	0.000	
	0.071	0.000	0.022	0.000	
	0.688	0.011	0.053	0.000	
	1.469	0.271	1.068	0.044	
	1.788	0.365	1.380	0.090	
	1.098	0.048	1.721	0.350	
	2.083	0.243	2.499	0.092	
	2.455	0.046	2.081	0.402	
	3.136	0.003	3.549	0.015	

Table 2aParameters for Wage and Home Production Equations
Estimated from NLSY
Allowing For Heterogeneity

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Log
Wage
0.095
0.119
0.010
0.028
0.134
0.011
0.136
-0.010
0.054
-0.332
0.022
0.000
-0.381
0.020
-0.084
-0.015
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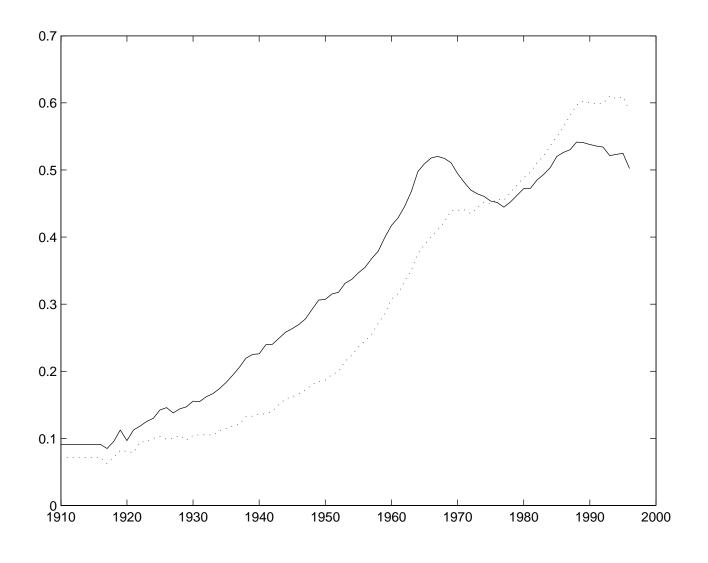
Table 2bParameters for Wage and Home Production Equations
Estimated from NLSY
Allowing For Heterogeneity

Figure 1 Estimated Rental Rate on High School and College Human Capital



Solid Line Rental Rate Male High School
Dotted Line Rental Rate Male College
Dashed Line Rental Rate Female High School
Dashed/Dotted Line Rental Rate Female College

Figure 2 Probability of Attending College by Cohort



Solid Line Men Dotted Line Women

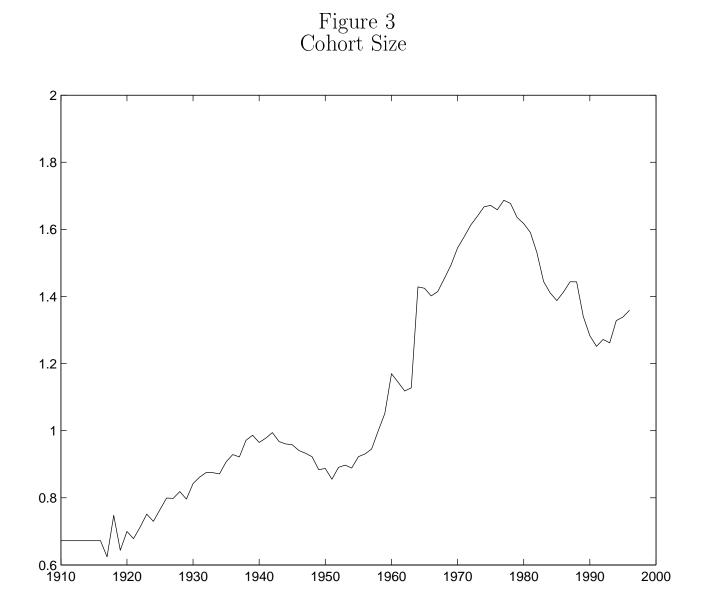
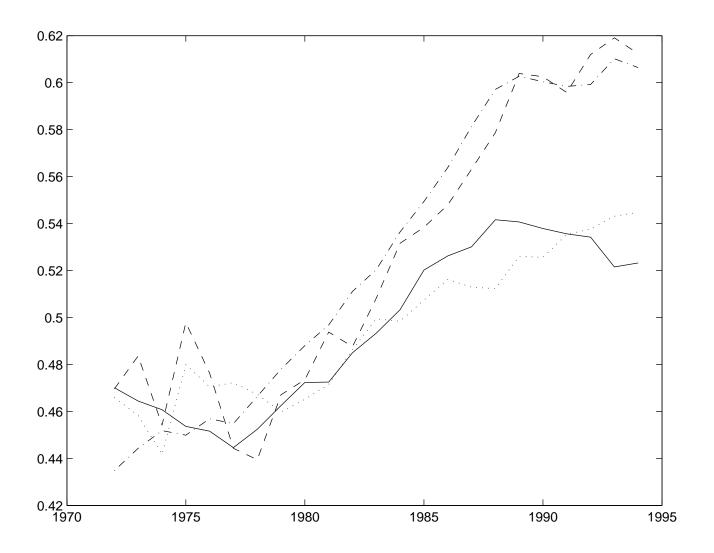


Figure 4 Predicted and Actual College Attendance Over Time



Solid Line Actual College Attendance: Men
Dotted Line Predicted College Attendance: Men
Dashed Line Actual College Attendance: Women
Dashed/Dotted Line Predicted College Attendance: Women

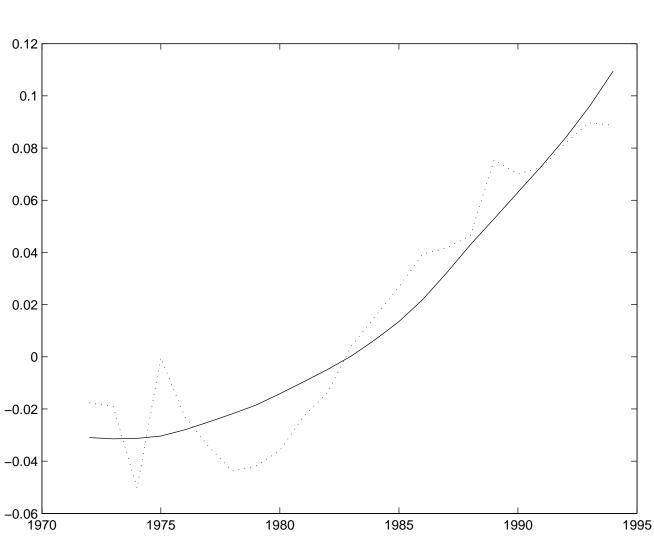


Figure 5 Predicted and Actual Log Price Differentials

